Photodynamic Notes, VI. By Pliny Earle Chase, LL.D.

(Read before the American Philosophical Society, October 6, 1882.)

242. *Stability of Harmonies.*

In Note 220, I presented several reasons for believing that the mean periods of planetary rotation are stable. They are all dependent upon more general principles which regulate the harmony of persistent oscillations in elastic media, and consequently furnish strong *a priori* presumptions against all hypotheses which seem, in any way, to conflict with harmonic tendencies. The certainty (Note 213), which Proctor admits, of Earth's having a pulsation period, with which its rotation must once have begun to approach to synchronism, springs from a like source with the harmonic tendencies in Jupiter's satellite system, and Laplace's reasoning is equally applicable to both cases. The "pulsation period" which is due to luminous vibration is constant, and if it should ever be suddenly or greatly disturbed, rotation would immediately begin again to approach to its normal synchronism. After the synchronism is once reached, all the influences from which it originated continue to contribute towards its perpetual maintenance.

243. *Improbability of Delaunay's Hypothesis.*

Newcomb and Holden (*Astronomy*, p. 148) close their note on the secular acceleration of the Moon, as follows :—"The present theory of acceleration is, therefore, that the Moon is really accelerated about six seconds in a century, and that the motion of the Earth on its axis is gradually diminishing at such a rate as to produce an apparent additional acceleration which may range from two to six seconds." The former portion is known to be cyclical, to be followed, after a long interval, by a corresponding retardation ; there is not a particle of evidence to discredit the probability that the latter portion is also cyclical. Neither is there a particle of evidence that there is any tidal friction except at the shores of the ocean, where any accelerating tendencies at one period are counterbalanced by retarding tendencies at another. The frictional hypothesis was a gratuitous assumption, to explain a doubtful phenomenon, and although the explanation would be satisfactory if the frictional retardation could be proven, the assumption violates the ordinary rules of framing scientific hypotheses so completely, that its chief claim for consideration rests upon the reputation of its originator. On the other hand, the harmonic hypothesis makes no assumption ; starting from acknowledged facts and principles, it asks what results may be reasonably anticipated, and there are few, if any, modern researches, in which the anticipations have been so abundantly verified. Even if we grant frictional retardation, there is "no way of determining the amount of this retardation unless we assume

that it causes the observed discrepancy between the theoretical and observed accelerations of the Moon " (*op. cit.* p. 147).

244. *Scientific Skepticism.*

Hesitation in the acceptance of alleged results, in any new line of scieatific research, is an obvious duty on the part of those who are fitted and expected to be on the watch against the promulgation of hasty generalizations which would needlessly cumber the field of knowledge. There is danger, however, that even faithful watchmen may sometimes hinder scientific progress by failing to keep their skepticism within proper bounds. The fact of harmony, and especially of coördinated harmony, transcends all mathematical tests of probability. It would be a tedious, but not a difficult task, to find in how many ways the letters of the Iliad could be arranged, and it is often wrongly assumed* that in a purely accidental arrangement of the letters, the faultless one would be as likely to take place as any other. It would be no more absurd to inquire whether the music of an orchestra might not be accidental, than to make a like inquiry as to the rhythm of atoms and waves and spheres. When mathematical tests confirm the probability that special forms of harmony are due to special laws, as in phyllotactic, thermodynamic and fundamental atomicities, they are useful ; but when they fail to give any reason for obvious accordances, as in Schuster's first examination of spectral lines (Note 141), they are utterly worthless unless they awaken further inquiries which lead to satisfactory results, as in Schuster's final conclusions.

245. *Centre of Dawning Condensation in the Terrestrial Belt.*

The intrinsic probability that the major axis of the Moon's orbit is invariable, is greatly enhanced by the following proportion :

$$r_3 : l_3 :: R_3 : L_3$$

Substituting the several known values, we have : r_3 = Earth's equatorial semi-diameter = 3962.8 miles ; l_3 = Laplace's terrestrial limit = $\left(\frac{86164.1}{5073.6}\right)^{\frac{2}{3}} r_3$; R_3 = Moon's semi-axis major = $60.2778\, r_3$; L_3 = limit of incipient belt-condensation = $R_3\, l_3 \div r_3$ = 1,578,217 miles. The oscillatory value of Sun's mass (Note 23, etc.,) gives, for the ratio of Earth's subsidence from the centre of the belt of greatest condensation, $L_3 \div \rho_3$ = 1,578,217 ÷ 92,785,700 = .0170093, and for the dawning central locus of the belt of greatest condensation, $1.0170093\, \rho_3$. The arithmetical mean between Stockwell's estimates of Mercury's secular perihelion and the secular aphelion of Mars is (.2974008 + 1.736478) ÷ 2 = $1.0169394\, \rho_3$. The difference between the two estimates is less than $\frac{1}{142}$ of one per cent

* See Note 252.

246. *Pendulum Estimate of Moon's Mass.*

In Note 8, I anticipated slight modifications of my first estimate of Moon's mass, as likely to be required by subsequent investigations. If we apply the principles which are involved in the coefficient of solar torsion, Note 162, to the determination of the length of Earth's theoretical pendulum, we find

$$l = g\left(\frac{t}{\pi}\right)^2 = \frac{32.088}{5280} \times (43082.05)^2 \div \pi^2 = 1,142,882 \text{ miles.}$$

From this equation we deduce the relative value of Moon's mass, μ, by the proportion,

$$\rho_3 : l :: m_3 : \mu$$
$$92,785,700 : 1,142,882 :: 81.1857 : 1$$

This estimate differs from the one in Note 8 by less than $\frac{1}{51}$ of one per cent.

247. *Rotation Estimate of Moon's Mass.*

The conviction, which I have often expressed (Note 220, etc.), that rotation is only modified revolution, is further strengthened by the following considerations. The orbital velocity (v_a) which the combined energies of Earth and Moon tend to give to an equatorial particle which is nearest to the Moon, is about 2.18 times as great as the velocity (v_β) which they tend to give to the mean centre of gravity of Earth's oscillating particles. The preponderating attraction of Earth prevents the action of these tendencies, in any other way than as accelerating disturbances on the several particles whose retarded and constrained revolution leads to axial rotation. The greater acceleration, acting for a half-monthly oscillation (t_a), gives the mean orbital velocity of the system (v_o), while the smaller acceleration, acting for a half-daily oscillation, gives Earth's equatorial velocity of rotation (v_r), as is shown by the proportion

$$v_o : v_r :: v_a\, t_a : v_\beta\, t_\beta$$
$$18.4735 : .288183 :: 14.7652942\, v_a : \tfrac{1}{2}\, v_\beta$$
$$v_a = 2.1798\, v_\beta.$$

If we designate the distances of the respective particles from the centre of gravity of the system by d_a and d_β, we have $d_\beta\, v_\beta^2 = d_a\, v_a^2$; $d_\beta = 4.7514\, d_a$. The theoretical mean intersections of d_a with Earth's surface should be on the equator, and those of d_β should be on meridians, but want of exact homogeneity, as well as orbital inclinations, may be presumed slightly to modify their respective loci. The mean centre of gravity of Earth's oscillatory particles is at the distance r from the surface, but they are all also affected by wave-lengths equivalent to d_a, so that we have $d_\beta = d_a + r = 4.7514\, d_a$. Hence $r = 3.7514\, d_a$; $d_a = .26657\, r = 1056.35$ miles; $d_\beta = 5019.15$ m.; $r - d_a = 2906.45$ m.; $m_3 + \mu = (238,869 \div 2906.45)\, \mu = 82.1858\, \mu$; $m_3 = 81.1858\, \mu$, a value which corresponds exactly with the one in the foregoing note.

248. *Harmonies of Central Condensation.*

The superficial intersections of d_a, in the foregoing note, describe circles about the poles, which have diametrical arcs of 5° 10′ 40″, which differs by only 2′ from the inclination of the Moon's orbit. If we take $1\times2\times3\times5$, the product of the first four phyllotactic numbers, as a divisor of Earth's diameter, calling the quotient a, we have the following approximate accordances:

Harmonic.	Observed,
$4\ a = 1056.748$ miles.	$d_a = 1056.35$ miles.
$11\ a = 2906.057$ "	$r - d_a = 2906.45$ "
$15\ a = 3962.805$ "	$r = 3962.8$ "
$19\ a = 5019.553$ "	$d_\beta = 5019.15$ "
$7\ a = 1849.309$ "	$r - 2\ d_a = 1850.10$ "

The coefficients of nodal division in the radius which is nearest the Moon, (4, 11), are the second and fourth of the secondary phyllotactic numbers. The coefficients in the remote radius, (8, 7), are the third phyllotactic numbers in the primary and secondary series, or the artiad and perissad divisors (Notes 201–2,). It may be interesting to inquire whether the frequency and locality of earthquakes are affected by these nodal influences.

249. *Pendulum Estimate of Earth's Oblateness.*

The ratio of Earth's equatorial semi-diameter to its theoretical equatorial pendulum, or the corresponding ratio of v_o^2 to v_r^2, (square of limiting orbital velocity to square of equatorial rotation-velocity), represents a centrifugal force which would tend to produce oblateness in a liquid globe, to maintain oblateness in a solidified globe, or to exert a constant pressure for restoring oblateness, should it be temporarily disturbed in any way. From the estimate of the theoretical pendulum in Note 246 we get

$$3962.8 : 1,142,882 : : 1 : 288.40$$

Bessels' estimate was 298.1528; Clarke's two estimates 291.36, 293.76; Listing's (1878, cited by Newcomb and Holden, p. 202), 288.5. This accordance furnishes additional reasons for believing that Earth's rotation and Moon's mean distance are as invariable as planetary major axes.

250. *Oscillatory Relations of Venus.*

The masses of Venus and Earth are more nearly alike than those of Jupiter and Saturn. This is perhaps owing to their comparatively central position in the belt of greatest condensation. The reasonable expectation that their mutual actions and reactions should be rhythmical is strengthened by many harmonic relations, among which are the following:

1. If we divide Venus's mean locus of subsidence (mean aphelion) by the product of the first four phyllotactic numbers, $1 \times 2 \times 3 \times 5 = 30$,

and call the quotient a, we obtain an approximate harmonic divisor for six cardinal nodes:

Harmonic.		Observed.		
$27\,a$	.6740	Venus,	s. p.	.6722
$28\,a$	.6990	"	m. p.	.6978
$29\,a$	.7239	"	m.	.7233
$30\,a$	.7489	"	m. a.	.7489
$31\,a$	.7739	"	s. a.	.7744
$40\,a$	.9985	Earth,	m.	1.0000

2. Venus's incipient locus of subsidence (secular aphelion) is near the second centre of linear oscillation of the incipient locus of subsidence of Mars.

$$(\tfrac{2}{3} \text{ of } \tfrac{2}{3} = \tfrac{4}{9}) \text{ of } 1.7365 = .7718.$$

Harmonic.	Observed.
.7718	.7744

3. The photodynamic origin of Venus's orbital period (224.701 days) is indicated by the proportion,

$$\rho_3 : l_\lambda :: t_e : t_v$$

The length (l_λ) of a theoretical pendulum at Sun's equator, which would oscillate once while a wave of light traverses the solar modulus of light, is $l_\lambda = 224.261\ \rho_3$; t_e and t_v are respectively Earth's day and Venus's year.

4. Moon's semi-axis major is a mean proportional between Earth's semi-diameter (r_3) and Venus's nearest approach to Earth. Venus's secular aphelion $= .7744234\ \rho_3$; Earth's secular perihelion $= .9322648\ \rho_3$; difference, $.1578414\ \rho_3 = 3695.725\ r_3$; $\sqrt{3695.725} = 60.792$.

5. Earth's oscillatory influence on Venus's mean subsidence is indicated by the proportion

$$r_3 : l_3 :: \rho_m : \rho_s$$

$$3962.8 : 1{,}142{,}882 :: 60.2778 : 17384.276$$

Stockwell's estimate for Venus's mean locus of subsidence is $.748878\ \rho_3 = 17534.36\ r_3$.

6. All the orbital loci of Venus are midway between Sun and orbital loci of Mars.

7. Venus's incipient rupturing locus (secular perihelion $= .672\rho_3$) is near Earth's linear centre of oscillation ($\tfrac{2}{3}$ of ρ_3.)

8. Venus's mass indicates Earth's harmonic influence at her incipient locus of subsidence (ρ_s).

$$m_3 : m_2 :: \rho_3 : \rho_s$$

$$428{,}417 : 331{,}776 :: 1 : .7744234$$

Hill's estimate for $m_0 \div m_2$ is 427,240, which differs from the harmonic estimate by less than $\tfrac{3}{10}$ of one per cent.

251. *Oscillatory Relations of Mercury.*

The cardinal loci of Mercury show the following among other harmonic relations:

1. The locus of Mercury's semi-axis major (.3871) is the rupturing locus for Venus's locus of incipient subsidence: ($\frac{1}{2}$ of $.7744 = .3872$).

2. Mercury's incipient rupturing locus (.2974) indicates phyllotactic influence at Venus's locus of incipient subsidence (.7744)

$$.2977 : .7741 : : 5 : 13.$$

3. Mercury's incipient rupturing locus (.2974) is also near the extremity of the linear pendulum, which has Mars's incipient subsidence locus (1.7365) for its point of suspension, and Venus's incipient subsidence locus (.7744) for its centre of oscillation:

$$(3 \times .7744 - 1.7365) \div 2 = .2934.$$

4. If we divide Earth's semi-axis major by the phyllotactic product $2 \times 3 \times 3 \times 13$, we find approximate indications of Earth's harmonic influence on Mercury's cardinal loci.

70 a	.298	Mercury	s. p.	.297
75 a	.320	"	m. p.	.319
91 a	.388	"	m.	.387
107 a	.456	"	m. a.	.455
112 a	.477	"	s. a.	.477
234 a	.997	Earth		1.000

252. *Improbability of Accidental Harmonies.*

Schuster's harmonic investigation (Note 141) appears to have been grounded on the hypothesis, which others have also entertained, that harmonies such as are found in spectral lines and planetary positions may be accidental. In note 244, I spoke of such an hypothesis as "wrongly assumed," and I believe that it is only calculated to hinder scientific progress. Professor Peirce, in the Howland will case, showed that the relation of each individual position to all the possible positions which it might assume, as well as the relative positions of the lines among themselves, should be considered in calculations of mathematical probability. In the Iliad problem, the bare improbability of the accidental arrangement of the letters in their orderly sequence is a^n, a representing the number of letters in the alphabet and n the number of letters in the poem. Let p be the number of readily distinguishable positions which each letter can assume, and the adverse probability against the accidental occurrence of the actual positions would be $(ap)^n$. The improbability would be likewise increased by considerations of the spaces between the letters, the word spaces, the orderly arrangement of lines and pages, the probable frequency of errors, and countless other particulars which are indicative of plan and purpose. Finally, the adequate explanation which is furnished by the simple hypothesis of human contrivance, wholly removes the question from the realm of chance, and makes the improbability infinite.

253. *Probability of Anticipated Results.*

It is not likely that any one would ever think of attributing the angles of crystals to accident, although it would not be so unreasonable to do so as it would be to account for much closer harmonies in that way. The laws of crystallization are obscure and almost wholly unknown, and yet we are not slow in believing that there are such laws, in spite of the irregularities which were pointed out in Note 232. The laws of elasticity, which lead to nodal action, are as well understood as any of the fundamental truths of physical science, yet there are many who fail to recognize them, and who seem to think that no explanation is needed of the harmonies which thrust themselves upon us on every hand. I am not aware that any attempt has ever been made, by any one who believes in the possibility that connected harmonies may be merely accidental, to confirm his belief by framing a series of such harmonies. In ordinary investigations, the discovery of a single fact, through anticipations which are grounded upon theoretical assumptions, is hailed as a wonderful scientific achievement. In the study of rhythmic elasticity such successful anticipations may be endlessly multiplied before their importance becomes generally understood. And yet each one of those verified anticipations lends a confirmation to the photodynamic hypothesis which is little, if any, short of absolute certainty, and which cannot be measured by any ordinary test of mathematical probability.

254. *A Photodynamic "Problem of Three Bodies."*

We have now gathered, by strictly Baconian methods, all the facts which are needed for framing and solving the following problem: To find simple stellar, planetary and satellite relations of mass, position and æthereal density, that will satisfy tendencies to the formation of three primary harmonic nodes, in an elastic medium which propagates undulations with the velocity of light.

1. Nodal tendencies presuppose some deviations from absolute homogeneity, which lead to differences of direction and velocity in the subsiding particles, thus giving rise to oscillations which continually incline to take some form of synchronism. As long as there is any liberty of motion among the particles, those which are at the boundary line, between the constraining inertia of central stellar nucleation and æthereal impulse, will oscillate with the greatest rapidity, tending to assume paths which will alternately receive and exhaust the projectile energies of the æthereal medium. Those energies cannot be completely exhausted until enough time has elapsed to communicate the velocity of light (v_λ), to an æthereal particle which is at rest at the beginning of the oscillation. The central inertia makes the oscillations circular, changing free elliptic revolution into constrained axial rotation, each oscillation of half-rotation occupying a time (t) which gives $gt = v_\lambda$; $gt_2 =$ modulus of light $= M$; $M \div \pi^2 =$

length of a theoretical pendulum, at the stellar equatorial surface, which would swing synchronously with the rotary oscillations; $g = \frac{v_\lambda}{t} = \frac{m}{r^2}$. The value of g determines the mean orbital velocity, $\sqrt{g r_n}$, for any semi-axis major, r_n.

2. The actions and reactions, between the stellar centre and the primary centre of planetary condensation (Note 23), involve tendencies towards the linear centre of gravity ($\frac{1}{2}$), the centre of linear oscillation ($\frac{1}{3}$), the centre of conical oscillation, ($\frac{1}{4}$), and centripetal accelerations which vary as the fourth power of the velocity of circular orbital revolution. These tendencies may all be satisfied by a stellar mass which is $(2 \times 3 \times 4)^4 = 331\text{-}776$ times the mass of primary condensation.

3. The orbital control of the stellar centre is exercised on the planet and satellite alike, at the mean distance ρ_3. If the planet transfers to the satellite a projectile *vis viva*, (l), corresponding to its superficial energy of rotation (Note 246), the relative masses of the planet and satellite, which satisfy their joint oscillatory relations and Sun's projectile energy, may be represented by the proportion :

$$\rho_3 : l :: m_3 : \mu.$$

255. *Subordinate Tendencies.*

There are other harmonic tendencies which seem likely to have been less permanent and more open to modification. The following instances of primitive tendency may be given as interesting:

4. The radii of static equlibrium are inversely as the masses; rupturing *vis viva* is acquired by subsidence through $\frac{1}{2}$ radius; if the rupturing locus of simple subsidence becomes a centre of linear oscillation for satellite semi-axis major, ρ_μ, we have

$$\rho_\mu : r_3 :: \tfrac{3}{2}\, m_3 : 2\, \mu.$$

5. The relations of æthereal density are found by the method of Note 240.

Notes 162, 23, and 246 give the following mass values which *precisely* satisfy the first three of these requirements, viz: $m_0 = 331{,}776\ m_3$; $m_3 = 81.186\ \mu$. The fourth requirement points to the value, $m_3 = 80.372\ \mu$. This slight discrepancy may, perhaps, be partly owing to the fact that Earth's oscillation is mainly rotational while Moon's is nearly that of a circular pendulum.

256. *Other Approximations to Moon's Mass.*

a. The formula, $mt^2 \propto \rho^3$, gives the following approximations to the value of μ: (1 year $\div$ 1 lunar mo.)$^2 = 178.724$; $(\rho_3 \div \rho_\mu)^3 = 58{,}609{,}000$; $(m_0 + m_3) = 327{,}930\ (m_3 + \mu) = 331{,}777\ m_3$; $m_3 = 86.241\ \mu$.

b. A close harmonic approximation is given by the proportion :

$$m_3 : \mu :: 6\, t_3 : t_\mu :: 2191.54 \text{ dy} : 27.32166 \text{ dy} :: 80.214 : 1.$$

c. The coefficient of t_3, in the above approximation, is the phyllotactic product, $1 \times 2 \times 3$. It is also very nearly equivalent to the square root of the quotient of Laplace's solar limit by Sun's semi-diameter, which would give, $m_3 = 80.619\ \mu$.

d. The mass of Mars is very nearly a mean proportional between the masses of Earth and Moon; $(3,093,500 \div 331,776)^2 = 86.938$.

e. An approximation similar to *b* is given by the proportion:

$$\text{month} : \text{day} :: m_3 : 3\ \mu :: 81.965 : 3.$$

f. Moon's locus of subsidence, or aphelion (s), and the mass of Venus (m_2), furnish the following approximation:

$$m_2 : \mu :: s : r_3 :: 63.593 : 1.$$

Substituting the observed basis of the second approximation to m_2 in Note 250, this gives, $m_3 = 82.119\ \mu$.

Many other approximations might doubtless be found which would represent obvious harmonic tendencies within the belt of greatest condensation.

257. *Simplicity and Conciseness of Harmonic Calculus.*

The range of estimates in the foregoing note is about $8\frac{3}{8}$ per cent., and the mean of all the estimates is about 2 per cent. greater than the most recent astronomical estimates. These deviations are four times as great as in my extreme estimates of solar distance, and twelve times as great as in the estimates which have been based upon the latest determinations of the harmonic elements. If these approximations are compared with those which had been made by astronomers, a hundred years after Newton had published the laws of gravitation, the indications of superiority in the harmonic methods become very striking. The difficulty of finding the harmonic influences which are most important, is incomparably less than that of determining the corresponding gravitating influences, and the saving of labor is obvious to every one who has ever solved astronomical problems by the ordinary processes of mathematical analysis. Doubts as to the degree of certainty which attaches to purely harmonic results will naturally arise, in the minds of those who have never carefully inquired into the necessity of elastic rhythm, but I believe that such doubts will gradually yield to the fast accumulating evidences of its universal sway. Astronomical, chemical and mechanical science may all be challenged to produce a series of connected fundamental determinations that are comparable, in precision and in intrinsic mathematical probability, with those which are embodied in Note 168 and in the three solutions of Note 254.

258. *Needless Obscurity.*

In Sir John Leslie's Dissertation on the Progress of Mathematical and Physical Science (*Encyc. Brit.*, 8th. Ed., i, 732), after referring to the "maze of intricate and abstruse *formulæ*" in which Laplace had involved the phenomena of capillary attraction, the following reflections of Dr.

Thomas Young are quoted :—"It must be confessed that, in this country, the cultivation of the higher branches of the Mathematics, and the invention of new methods of calculation, cannot be too much recommended to the generality of those who apply themselves to Natural Philosophy ; but it is equally true, on the other hand, that the first mathematicians on the Continent have exerted great ingenuity in involving the plainest truths of mechanics in the intricacies of Algebraical formulas, and in some instances have even lost sight of the real state of an investigation, by attending only to the symbols, which they have employed for expressing its steps." After this quotation Leslie proceeds as follows :—"Laplace's intricate formula has been since unraveled by the acute discrimination of Mr. Ivory, who disjoined it into two separate portions ; the one depending on the adhesion of the watery film to the inside of the tube, and the other resulting from half the cohesion of the particles of the liquid to each other. But our ingenious countryman deduced these elements of the complete force from the simplest physical principles, availing himself of the property of equable diffusion of pressure through the mass of a fluid. The same investigation gave the measure and limits of depression observed in mercury and some other liquids."

259. *Coöperative Methods.*

Since the invention of Hamilton's quaternions and Peirce's linear associative algebras, the temptation for mathematicians to involve "the plainest truths of mechanics in the intricacies of algebraical formulas" has greatly increased. The higher the algebra, the smaller is the number who are able to understand it. While it may be no part of an investigator's duty to "popularize" science, no result can be rightly regarded as belonging to the dominion of science until it has been so far popularized as to be brought within the grasp of the majority of scientific men who are willing to follow the several steps of the original investigation. Labor which is expended on intricate solutions of problems which can be simply deduced from "the property of equable diffusion of pressure through the mass of a fluid," or from other properties of elastic media, is either labor wholly wasted, or, at best, an exercise of ingenuity which serves as a harmless recreation. On the other hand, the use of well-known physical relations as clews for the discovery of coördinate relations, alternating with analytical solutions of problems which are suggested by such discoveries, combines the advantages of theory and observation in ways which are most helpful to scientific progress. Whenever any given result may be reached by two or more different methods, the shortest and simplest is always most commendable.

260. *Lunar Magnetic Polarization.*

The relations between magnetic fluctuations and gravitating tendencies to the restoration of equilibrium in disturbed atmospheric or æthereal currents

(Notes 116–122), are, as might reasonably have been foreseen, greatly modified by Sun's thermal activity. The moon, acting on the currents which originate in Sun's thermal disturbance, shows accordances both in time and magnitude (Note 121), which point strongly, if not conclusively, to an absolute identity between lunar disturbances of terrestrial magnetism and of terrestrial gravitation. These pointings are confirmed by the identity of velocity, in the electro-magnetic "ratio," in the pendulum-oscillations of solar rotation, and in the transmission of luminous undulations. The symmetrical arrangement of æthereal particles which most simply represents the results of elastic pressure (*Proc. Amer. Phil. Soc.*, xii, 408), the spiral tendencies of division in extreme and mean ratio, the rotation which helps to maintain equilibrium between conflicting forces (Note 212), the differences of centrifugal and centripetal energy which result from rotation, all contribute towards an axial polarity which should modify all forms of chemical and mechanical aggregation. To these elements of cyclical rhythm Moon adds her orbital disturbance of Earth's rotation (Notes 247–8), which is so modified by orbital inclination as to produce a magnetic nutation. If we add to these considerations the oscillations of Earth's crust, and other influences which lead to variations in the relative positions of areas of greatest heat and cold, we find data for many interesting problems in mathematical analysis, the solution of which may throw much light both on the normal and abnormal phenomena of terrestrial magnetism.

261. *Gravitating Modulus of Planetary Revolution.*

The hypothesis that stellar rotation is merely retarded revolution, and the exact correspondence between the time of rotary oscillation and the time in which maximum gravitating acceleration would communicate the velocity of light to an æthereal particle, suggest the likelihood of other moduli, which may be intimately connected with the solar-equatorial modulus of light, and which may help us towards a fuller understanding of fundamental kinetic relations. As the rotary oscillations are circular, the simplest and most natural comparison would refer them to circular revolutions of uniform velocity; as all orbital times and velocities are functions of mass and distance, it seems right to begin by examining the greatest possible limit of circular-orbital velocity ($\sqrt{g_0 r_0}$), and the least possible limit of circular-orbital oscillatory time $\left(\frac{1}{2} t \text{ of revolution} = \pi\sqrt{\frac{r_0}{g_0}}\right)$. The British Nautical Almanac value of n, Note 75, gives $t = \frac{1}{2}\, t_0 = 5024.5$ sec.; $g_0\, t = \pi \sqrt{g_0\, r_0} = .0019643\; r_0$; $g_0\, t^2 = \pi^2\, l = \pi^2\, r_0$. The photodynamic relations of this fundamental gravitating modulus, to the two chief planetary loci, are shown by the approximate identity of t with the time in which a luminous ray would traverse Jupiter's orbit or Saturn's mean aphelion radius vector. Neptune's gravitating modulus, $\pi^2\, \rho_8$, represents

the second supra-Neptunian locus which I indicated in 1873, and which Forbes found to represent a group of cometary aphelion distances (Note 32).

262. *Photodynamic Modulus of Planetary Revolution.*

A mathematical friend, in whose judgment I place great confidence, admits the conclusiveness of the evidence in favor of paraboloidal harmony in rupturing planetary loci (Note 46, etc.), but he thinks that the approximation to the locus of *Alpha Centauri* may be accidental. I am well aware of the difficulty, which every one naturally finds, in believing that the seemingly quiet undulations of light should have any influence on the relative positions of stellar systems. The remembrance that the *vis viva* of action or reaction, for any given mass, varies as the square of oscillating velocity, would show that if there is any physical influence which controls interstellar arrangements, it should be the one which has the greatest normal velocity. The parabolic energy which is manifested within the solar system, both in approaching and in leaving the sun, must be indefinitely extended, and the luminous undulations which it indicates are equally extensive. The symmetry of the three-fold division in the paraboloid, together with the fact that the uncertainty of stellar distance is of the same order of magnitude as planetary eccentricities, excludes any probable attribution of the stellar accordance to accidental coincidence. The foregoing note furnishes additional grounds for accepting all the harmonic relations of the photodynamic paraboloid as effective. Since $L_0 \propto r^2$ (Note 75), and L_0 at $r_0 = \pi^2 l_0^3$, at the gravitating modulus $\pi^2 r_0$, the photodynamic modulus would be $\pi^6 l^3_0$, its logarithm being $6 \times .4971499 + 3 \times 1.5606934 = 7.6649796$. This is only .0013506 less than the logarithm for the locus of α *Centauri*, as deduced from the corona line and the British estimate of Sun's semi-diameter, indicating a difference of less than $\frac{5}{16}$ of one per cent. It is, therefore, *certain* that the photodynamic modulus of Sun's gravitating modulus is in the *neighborhood*, if not in the actual locus, of the nearest known star.

263. *A Chain of Photodynamic Harmonies.*

If we designate the planetary locus which corresponds to the corona line (Note 45) by x; Jupiter's greatest eccentricity by y; and the theoretical locus of α *Centauri* by z, the following connected equations can all be deduced from simple and obvious forms of elastic rhythm:

1.	$y = 1 - (1048.875 \div 5.202798\, n) = .06055$	Note 3
2.	$m_0 = (2 \times 3 \times 4)^4\, m_3 = 331776\, m_3$	" 23
3.	$x = \pi^{\frac{2}{3}}\, n r_0 = 460.61\, r_0$	" 45
4.	$z : x :: m_5^2 : m_3^2 \quad z = 461746300\, r_0$	" 46
5.	$v_0 = 2\pi\, r_0\, n^{\frac{3}{2}} \div 1 \text{ year} = .0006265013\, r_0$	" 75
6.	$V_0 = \pi\, v_0\, (l_0 \div r_0)^{\frac{3}{2}} = .4313442\, r_0$	" 75
7.	$V_0 = n r_0 \div 497.827 \quad n = 214.735$	" 75

8.	$z = \pi^6 \, l_0^3 \div r_0^2 \quad l_0 = 36.3497 \, r_0$	Note 261
9.	$r_a = r \div n = 960''.556$	" 75
10.	$r_0 = (m_0 \div m_3)^{\frac{1}{3}} \times (1 \text{ yr.} \div t_3)^{\frac{2}{3}} \times r_3 \div n = 432094$ miles	" 75
11.	$p_0 = r_3 \, r_a \div r_0 = 8''.8094$	" 75
12.	$t_0 = 1 \text{ yr.} \div n^{\frac{3}{2}} = 10029$ sec.	" 75
13.	$d_0 \div d_3 = (t_3 \div t_0)^2 = .255927$	" 75
14.	$L_0 = (V_0 \div v_0)^2 \, r_0 = 474028 \, r_0$	" 75
15.	Corona line $= 7612 \times \log. \, 30.037 \, n \div \log. \, x = 5321.7$	" 45
16.	$\rho_3 = nr_0 = 92,785,700$ miles	" 75

The values of y, m_0, x, z, l_0, L_0, and the corona line, all represent photodynamic considerations; the other values are readily deduced from them by simple radiodynamic relations. Stockwell's estimate of y is .06083; the value of n is intermediate between those of the British and the American Nautical Almanacs; the value of the corona line corresponds *precisely* with the geometrical wave-length in Note 41; all the other values are within the astronomical limits of probable error.

264. *Further Oscillatory Relations of Venus.*

It seems not unlikely that the position of Venus, in the belt of greatest condensation, may have nearly as many suggestive relations as that of Earth. To the eight indications of Note 250, the following may be added:

9. All the orbital loci of Venus are between a primary and a secondary centre of linear oscillation for Earth's semi-axis major ($\frac{2}{3}$, and $\frac{2}{3} + \frac{1}{3}$ of $\frac{1}{3}$ $= \frac{7}{9}$). Stockwell's estimates are, secular perihelion, .6722; secular aphelion, .7744.

10. The secular aphelion of Venus is nearly a mean proportional between Earth's second reciprocal centre of oscillation ($\frac{1}{3}$ of $\frac{1}{3}$), and Jupiter's secular aphelion; $\sqrt{\frac{1}{9} \times 5.42735} = .7766$.

11. The major-axis of the nebular ellipse which marks the incipient separation of Venus from Earth, 1.7744, is indicative of a successive nucleation for Earth's semi-axis major; $\frac{4}{3} \times \frac{4}{3} = 1.7778$.

12. The mass harmony (8), introduces the principle of virtual velocities into the foregoing nebular ellipse, at the beginning of subsidence for Venus.

265. *Tidal Harmony.*

The tidal disturbance of Earth by Sun, during a semi-annual orbital oscillation, is sufficient to give orbital velocity to all the particles which are disturbed both by Sun and by Moon. Orbital velocity would be communicated in $\frac{1}{\pi}$ of an oscillation, to the particles which are disturbed by Sun's tidal action. During the remainder of the oscillation a like velocity would be communicated to $\pi - 1$ times as many particles. If we desig-

nate Moon's mass and semi-axis major by m_a and ρ_a, this approximation gives us the following proportion :

$$m_a : \rho_a^3 : : (\pi - 1)\, m_0 : \rho_3^3.$$

Substituting $m_0 = 331776\ m_3$, $\rho_3 = 92785700$ miles, $\rho_a = 238869$ miles, we get, $m_3 = 82.486\ m_a$. The values which were found in Notes 8 and 246 seem likely to be subjeet to fewer modifications than this, but every additional indication of approximation to anticipated harmonies lends new interest to the discussion of elastic influence and furnishes new material for future analytic research.

266. *Harmonic Tidal Cycles.*

The tendency of the solar and lunar tidal disturbances to cyclic harmony, is shown by the approximate equality of the solar disturbance, during the interval which would give terrestrial particles orbital velocity, to the lunar disturbance, during a sidereal revolution about the Earth. The approximation may be expressed by the equation :

$$\frac{m_0}{\rho_3^3} \times \frac{1\ yr}{2\pi} \doteq \frac{m_a}{\rho_a^3} \times 1\ mo.$$

Substituting the same values as in the foregoing note for m_0, ρ_3 and ρ_a, we get the approximate value, $m_3 = 83.025 m_a$. The closeness of these various approximations may be attributed, with great likelihood, to original influences of central-belt condensation, aided by the natural stability of harmonic oscillations which have once been set up. The slight discrepancies between different estimates are probably owing to subordinate rhythmic disturbances, such as nutation, precession, and other oscillations, the exact influence of which we may reasonably hope to understand when we have a fuller knowledge of æthereal elasticity.

267. *Subterranean Tides.*

My views regarding the influence of elasticity upon tidal adjustments, (*Proc. Amer. Ph. Soc.*, ix–xiv ; xvi, sq.; *Phot. Notes* 215–8), are confirmed by the subterranean tides in the flooded mines at Dux, in Bohemia. In a communication to *Ciel et Terre* (copied in *Ann. de Chim. et de Phys.*, xxv, 533–46), M. C. Lagrange cites the discussion, by Grablowitz (*Boll. della Soc. Adr. di Sci. Nat. in Trieste,* vol. vi, fasc. I, 1880), of Klönne's observations. The observations seem to show conclusively that the ebb and flow in the mines is due to combined solar and lunar action, but that it can be satisfactorily explained only by the direct attraction of the two bodies upon the solid mass of the Earth. Lagrange refers to previous investigations, by himself and by George H. Darwin, which go to show that if cosmical bodies have any elasticity, they must undergo continual and periodic changes of form. Grablowitz infers that those changes should lead to oscillations of various intensity, so as to produce mechanical effects which differ according to the nature and degree of local elasticity,

but subjected to invariable laws which are regulated by the relative movements of the disturbing bodies. Naumann's tables (*Handbuch der Chemie*, 1877, pp. 346–59), show that if the whole Earth was a solid diamond, or if it was composed of rocks which are least expansible, the greatest quarter-daily tidal deformations would not involve an amount of work equivalent to $\frac{1}{2}^{\circ}$ C. The spring tidal stress during six hours is $\left(\frac{m_o}{\rho_3^{\,3}} + \frac{m_a}{\rho_a^{\,3}}\right)\frac{gt^2}{2} \doteq 619$ ft., which is enough to furnish many times the available force requisite for all the adjustments of æthereal elasticity, freely moving particles, and internal work in the solid rocks.

268. "*Conservation of Solar Energy.*"

The views of Dr. C. William Siemens suggest a consideration of the influence of solar rotation upon the æthereal atmosphere, at various distances from Sun's centre. Laplace's limit, according to the data in Note 263, is at 36.35 r_0. The centrifugal force of rotation at that limit would be $36.35^2 = 1321.3$ times as great as at Sun's surface, while the centripetal force of gavitation is only $\frac{1}{1321.3}$ times as great. The photographs of the solar eclipse which have been lately published (*Nature*, April 20th, 1882), indicate an atmospheric oblateness which may be due to the equilibrating tendencies of these two opposing forces. If the æthereal disturbances which result from this source are not sufficient to account for luminous and thermal vibrations, we may look still further to the velocity which the subsiding particles would acquire in falling from the equatorial limit to the poles. If there was no resistance, this velocity would be $\left(\frac{35.35}{36.35} \times 2gr\right)^{\frac{1}{2}} = 376.8$ miles per second. Any diminution of this velocity by resistance would be converted into heat. If the mean limit between the centrifugal and centripetal tendencies is in latitude 30°, the mean diminution of velocity when the particles reach the polar zone, would be .982 of 376.8 = 370 miles. If the mean time of accomplishing the centrifugal and centripetal cycles is the same as the time of half-rotary oscillation, the formula of torsional elasticity (Note 162) provides for radiations with the oscillatory velocity of light, and the general tendency of nebulæ to a discoid or flattened form gains a new significance.

269. *Another Test of Atomic Divisors.*

In order to avoid all questions of absolute probability, in Notes 171, 201, 202, etc., I have computed $(n\,D - O) \div D$ for all the elements in Clarke's table except H, using $D_1 = 7$ for the perissads, $D_2 = 8$ for the artiads, $D_3 = 1$ for the hydrogen divisor. Adding the logarithms of $(n\,D - O) \div D$, I find for the perissads,

Σ (for D_1) = A	$\overline{20}.4692966$
Σ (for D_3) = B	$\overline{22}.9326580$
A — B	1.5366386

The aggregate probability of the hydrogen divisor is, therefore, 34.406 times as great as that of the general perissad divisor, 7.

For the artiads

Σ (for D_2) $=$ C	$\overline{45}.3906748$
Σ (for D_3) $=$ D	$\overline{38}.1502848$
D — C	6.7596100

The aggregate probability of the general artiad divisor, 8, is therefore 5749234 times as great as that of the hydrogen divisor.

For all the elements,

Σ (for D_1 and D_2) $=$ E	$\overline{65}.8599714$
Σ (for D_3) $=$ F	$\overline{59}.0829428$
F — E	5.2229714

The aggregate probability of the atmospheric divisors is, therefore, 167098 times as great as that of the hydrogen divisor.

Dividing the sums of the perissad, artiad and total logarithms by 20, 44, 64, respectively, we get for the mean values of (n D — O) $\div$ D, and for the mean relative probability of phyllotactic influence,

		Log.	Antilog.	Probability.
Perissad,	D_1	$\overline{1}.0234648$	.10555	2.145
	D_3	$\overline{2}.9466329$	.08844	3.223
Artiad,	D_2	$\overline{2}.9861517$	.09686	2.124
	D_3	$\overline{1}.1397792$	.13797	1.491
Total,	D_1, D_2	$\overline{2}.9978121$	.09950	2.009
	D_3	$\overline{1}.0794210$	.12007	1.664

The relative probability is found by dividing the mean accidental ratios for 20, 44, and 64 numbers, with differences equally distributed, by the antilogarithms, or observed ratios. The accidental ratios are .22607 for the perissads, .20578 for the artiads, .19985 for the whole list of elements.

Some criticisms have been made upon my previous estimates of probability, which overlooked my demonstration that ordinary tests fail to show probabilities which are known to exist (Notes 145, 149), and my introduction of "the *a priori* probability of tendency to division in extreme and mean ratio" (Note 171). As my object is to show the *relative* probability of different divisors, and as it is impossible to know what weight should be given to *a priori* considerations, the present method may be acceptable.

270. *Fundamental Centrifugal and Centripetal Mass-Relations.*

The influence of cardinal loci upon the relative masses at the chief centre of nucleation and at the chief centre of condensation, is shown by the equation :

$$\frac{m_3}{m_0} \cdot \frac{l^4}{r_0{}^4} = \frac{\rho_5}{\rho_3}. \qquad (1)$$

In this equation, $\rho_3 =$ Earth's semi-axis major $= 1$; $\rho_5 =$ Jupiter's semi-axis major $= 5.202798$; $r_0 =$ Sun's semi-diameter ; $l =$ Laplace's solar limit $= 36.3658\, r_0$ (See Note 75). This gives for Sun's mass, $m_0 =$

336,153 m_3, which is 1.32 per cent. greater than the estimate which is based on requirements of oscillation and subsidence (Notes 5, 23, etc.). If f_0, f_1 designate the centrifugal force of rotation at r_0, l, respectively, and g_0, g_1 represent the corresponding centripetal accelerations of gravity, equation (1) may assume the form:

$$\frac{m_3}{m_0} \cdot \frac{f_1}{g_1} = \frac{\rho_5}{\rho_3} \cdot \frac{f_0}{g_0} \qquad (2)$$

Equation (1) is especially interesting for its bearing on the conservation of solar energy (Note. 268); equations (1) and (2) represent the equal ratios of action and reaction between centripetal and centrifugal tendencies, all the numerators having a centrifugal origin, while all the denominators are centripetal. Combining these equations with the equation of Earth's photodynamic *vis viva* (Note 91), we get

$$\frac{m_5}{m_3} \cdot \frac{l^4}{r^4} = \left(\frac{v_\lambda}{v_n}\right)^2 \cdot \frac{\rho_5}{\rho_3} \qquad (3)$$

Here also we have centrifugal numerators and centripetal denominators, together with photodynamic orbital relations of mass, distance, velocity, rotation, revolution and condensation, which are very suggestive.

271. *Perissad Relations of Nitrogen.*

If we take the continued product, for all the elements, of the percentages of D which represent $(n\ D - O) \div D$, the hydrogen product is 69.208 times as great as for Gerber's empirical divisors, and 18178.47 times as great as that for my phyllotactic factors (Note 136). While this is sufficient to show the influence of phyllotactic tendencies, my comparisons of relative probability have led me to the discovery of important modifications of these tendencies by the abundant gases, H, N, O; H being a representative factor of the monatomic elements, $\frac{1}{9}$ N for the tri- and pentavalent, 8 H for the di- and tetratomic, $\frac{1}{16}$ O = .998 for the remaining metallic elements. The effect of a slight difference in the divisor upon the residuals, as well as my method of operation, may be illustrated by testing Gerber's divisor ($D_1 = 1.559$) and my own ($D_2 = \frac{1}{9}$ N = 1.558) on the tri- and pentavalent elements:

	Clarke.		R_1.		R_2.	Log. R_1.	Log. R_2.
N	14.021	= 9 D_1	− 10	= 9 D_2	− 1	1.0000000	.0000000
P	30.958	= 20 D_1	− 222	= 20 D_2	− 202	2.3463530	2.3053514
As.	74.918	= 48 D_1	+ 86	= 48 D_2	+ 134	1.9344984	2.1271048
Sb	119.955	= 77 D_1	− 88	= 77 D_2	− 11	1.9444827	1.0413927
Bi	207.523	= 133 D_1	+ 176	= 133 D_2	+ 309	2.2455127	2.4899585
Au	196.155	= 126 D_1	− 279	= 126 D_2	− 153	2.4456042	2.1846914
Bo	10.941	= 7 D_1	+ 28	= 7 D_2	+ 35	1.4471580	1.5440680
Ta	182.144	= 117 D_1	− 259	= 117 D_2	− 142	2.4132998	2.1522883
V	51.256	= 33 D_1	− 101	= 33 D_2	− 158	2.2810334	2.1986571
Σ log. R						18.0579422	16.0435122
9 × log. D						28.7356149	28.7331075
Σ log. (R ÷ D) = log. P						$\overline{11}.3223273$	$\overline{13}.3104047$
Mean log. = $\frac{1}{9}$ Σ log. = log. p						$\overline{2}.8135919$	$\overline{2}.5900450$

The logarithm of aggregate relative probability is $\overline{11}.3223273 - \overline{13}.3104047 = 2.0119226$; the log. of mean relative probability is $\overline{2}.8135919 - \overline{2}.5900450 = .2235469$. Hence the aggregate relative probability of the nitrogen divisor, $P_2 \div P_1 = 102.783$; the mean relative probability, $p_2 \div p_1 = 1.6732$.

272. *Aggregate and Mean Ratio of Residuals to Atomic Divisors.*

In the following table the logarithms for each group are computed after the method of the foregoing note. The divisor for the first surd, S_1, is $\frac{1}{2}(3-\sqrt{5}) = .382$; for the second surd, S_2, $\frac{1}{2}(\sqrt{5}-1) = .618$; for hydrogen, H = 1; for Gerber and Chase I, see Note 136; for Chase II, see Note 269; for Chase III, see Note 271.

Group.	S_2.	S_1.	H.	Gerber.	Chase I.	Chase II.	Chase III.
Monat.	$\overline{8}.3393239$	$\overline{8}.5143370$	$\overline{12}.2138579$	$\overline{11}.7186094$	$\overline{11}.6283038$	$\overline{11}.4019386$	$\overline{12}.2138579$
3 and 5.	$\overline{7}.4310096$	$\overline{8}.7210766$	$\overline{10}.7187995$	$\overline{11}.3223273$	$\overline{11}.3223273$	$\overline{9}.0673580$	$\overline{13}.3104047$
2 and 4.	$\overline{11}.7492826$	$\overline{14}.6963491$	$\overline{16}.5909435$	$\overline{23}.8649303$	$\overline{23}.6476134$	$\overline{23}.3406987$	$\overline{23}.3406987$
Metal.	$\overline{18}.5073145$	$\overline{19}.1410255$	$\overline{23}.5593397$	$\overline{18}.3369171$	$\overline{20}.2251388$	$\overline{22}.0499751$	$\overline{32}.5388218$
Periss.	$\overline{15}.7703335$	$\overline{15}.2354136$	$\overline{22}.9326574$	$\overline{21}.0409367$	$\overline{22}.9506311$	$\overline{20}.4692966$	$\overline{25}.5242626$
Artiad.	$\overline{28}.2565971$	$\overline{33}.8373746$	$\overline{38}.1502832$	$\overline{40}.2018474$	$\overline{43}.8727522$	$\overline{45}.3906738$	$\overline{55}.8795205$
Aggreg.	$\overline{42}.0269306$	$\overline{47}.0727882$	$\overline{59}.0829406$	$\overline{61}.2427841$	$\overline{64}.8233833$	$\overline{65}.8599704$	$\overline{79}.4037831$
Mean.	$\overline{1}.3441708$	$\overline{1}.2667623$	$\overline{1}.0794209$	$\overline{1}.0506685$	$\overline{1}.0128654$	$\overline{2}.9978120$	$\overline{2}.7719341$
Rel. Ag.	.0000000	4.9541424	16.9439900	18.7841465	21.2035473	22.1669602	36.6231475
Rel. M.	.0000000	.0774085	.2647499	.2935023	.3313054	.3463588	.5722367

The aggregate residual ratio for S_2 is more than 87,900,000,000,000,000 times as great as for hydrogen, and more than 4,199,000,000,000,000,000,000,000,000,000,000 times as great as for the relations to H, N, O; the mean ratio is 1.8397 times as great as for H, and 3.7345 times as great as for my second group of divisors. The aggregate hydrogen ratio is more than 47,770,000,000,000,000,000 times as great as for my third group, the mean ratio being 2.0299 times as great.

273. *Comparison of Geometric and Arithmetic Residual Means.*

The logarithms of the geometric mean residual ratios, for the several groups, may be found by dividing the monatomic logarithms by 11, the tri- and pentatomic by 9, the di- and tetratomic by 17, the metallic by 27. Some questions of relative probability may be tested more readily by arithmetical means, and for this reason as well as in order to preserve additional evidence of phyllotactic influence, the following table is given. All of my divisors were deduced from phyllotactic considerations; the first set shows the great superiority of my phyllotactic over Gerber's approximately phyllotactic divisors; the second set introduces corresponding terms of two phyllotactic series; the third set has two divisors which are simply phyllotactic (1, 8) and two which are products of phyllotactic ratios ($\frac{1}{16} = \frac{1}{2} \times \frac{1}{8}$; $\frac{1}{9} = \frac{1}{3} \times \frac{1}{3}$).

Group.	Arithmetical. S_2.	S_1.	H.	G.	C_1.	C_2.	C_3.	Geometrical. S_2.	S_1.	H.	G.	C_1.	C_2.	C_3.
Monat.	.237	.232	.215	.195	.171	.205	.215	.201	.209	.085	.116	.114	.109	.085
3 and 5.	.218	.225	.142	.096	.096	.186	.082	.186	.155	.093	.065	.065	.102	.039
2 and 4.	.288	.221	.214	.144	.145	.137	.137	.249	.165	.124	.050	.048	.046	.046
Metal.	.268	.259	.225	.246	.240	.227	.127	.225	.200	.148	.222	.185	.154	.068
Periss.	.229	.228	.182	.150	.137	.172	.155	.194	.183	.088	.090	.089	.106	.060
Artiad.	.276	.245	.220	.207	.203	.192	.131	.234	.186	.138	.125	.110	.097	.059
Aggreg.	.261	.240	.209	.189	.183	.186	.139	.237	.185	.120	.112	.103	.099	.059

274. *Some Consequences of the Identity of Luminous and Gravitating Oscillation.*

The æthereal particles, which are repelled from Sun's equator by the centrifugal force of rotation, "subside" spirally towards the poles, giving rise to Ampérian currents which account for Maxwell's identification of luminous and electromagnetic waves and yielding a mechanical equivalent of 76,000,000 J for every pound of subsiding matter; the axial core of the spirals is the rod of the virtual solar pendulum (Note 162) of which the length and the radius of torsion are both determined by the solar modulus of light; the continual succession of spiral impulsions substitutes uniform rotation for reciprocal oscillation; the *precise* accordance between the time of rotary oscillation and the time of acquiring or losing the velocity of luminous projection, shows the equally precise agreement between centrifugal æthereal action and centripetal gravitating reaction; the combination of axial rotation with orbital revolution produces continual shiftings of inertial resistance which must be followed by continual renewals of æthereal disturbance; the perpetual maintenance of luminous oscillation by influx, as well as by an equivalent efflux, removes "the reproach of Thermodynamics."

Such are a few of the obvious considerations which are suggested by the identity of luminous and gravitating oscillatory velocity at the centre of the solar system. In subjecting them to the tests of mathematical analysis, the equilibrating tendencies of centrifugal and centripetal action should be studied with especial reference to three oblate spheroids, all of which have the same poles as the Sun. Their equatorial loci are respectively coincident with Laplace's limit ($36.35r_0$), the virtual radius of solar torsion ($688.95r_0$), and the solar modulus of light ($474657r_0$).

275. *Consideration of Some Objections.*

Professor Geo. Fras. FitzGerald (*Nature*, xxvi, 80) presents four questions in the way of objection to the hypothesis of the conservation of solar energy by an average influx which is exactly equivalent to the average efflux. The reply of Dr. Siemens may be supplemented by some additional considerations. "1. How the interplanetary gases near the Sun acquire a sufficient radial velocity to prevent their becoming a dense atmosphere around him?" The proportionality of centrifugal force to mass would combine with the tendencies of gaseous diffusion and with the in-

creased molecular velocity of gaseous condensation, to maintain a constant circulation of all the constituents of the solar atmosphere. "2. Why enormous atmospheres have not long ago become attached to the planets, notably to the Moon?" The reported discovery of a lunar atmosphere by Trépied and Thollon, during the late solar eclipse, which is announced in the same number as the question, gives an apt and timely confirmation of Wollaston's views, which are cited by Siemens. "3. Why the earth has not long ago been deluged when a constant stream of aqueous vapor that would produce a rain of more than 30 inches per annum all over the earth, must annually pass out past the earth in order to supply fuel to be dissociated by the heat that annually passes the earth?". The average annual rainfall of the whole globe is not accurately known, but there is good reason to believe that it is very nearly, if not precisely, such as would be required by the hypothesis. "4. Why we can see the stars although most of the solar radiations are absorbed within some reasonable distance of the Sun?" The prevalent thermodynamic hypotheses suppose an unlimited power of absorbing radiant *vis viva*, without any tangible evidence of such absorption. All that needs to be explained is the maintenance of a uniform amount of æthereal oscillations in the universe. If the centrifugal and centripetal alternations of less elastic particles are linked with like alternations of more elastic particles, the actions and reactions of elasticity and inertia may account for the operation and stability of all physical laws.

276. *Influence of Explosive Oscillations on Radiant Energy.*

The relations which I have pointed out between explosive oscillations and planetary positions (*Proc. Amer. Phil. Soc.*, xii, 392–417, *et seq.*), should also influence the centrifugal and centripetal alternations within Sun's photosphere. I have often shown that the photodynamic equality of luminous and gravitating oscillations tends to drive all of Sun's particles towards the limit between aggregation and dissociation, so that a slight external disturbance may turn the unstable equilibrium in either direction. Berthelot's investigations (C. R. xciii. 613–9) of explosions in gaseous compounds by detonating agents, have indicated the existence of explosive waves which are quite distinct from simple waves of sound, and have shown that compounds and explosive mixtures generally become more sensitive to shocks as they near the temperature at which they begin to decompose. Hence a meteor, or even a single molecule, which has acquired a sufficient velocity of subsidence in its sunward fall, may explode a gaseous compound which is in or near the nascent state, and the explosion may react upon the falling mass or molecule so as to leave it in an unstable equilibrium which is ready for explosion by the next like subsidence. The locus of the virtual radius of solar torsion (ar_o, Note 162) in the asteroidal belt, makes the minor planets important outposts of explosive oscillation in the second of the oblate spheroids to which I called atten-

tion in Note 274, and opens a wide field for analytical research respecting the equilibrating tendencies between centripetal and centrifugal energy. The stability of major axes, in orbital revolution, is no more necessary than the reciprocal equality of radiodynamic action and reaction in elastic media.

277. *Relations of Earth to the Asteroids.*

Different portions of the asteroidal belt are so related to Earth's varying positions as to share the influence, either directly or through the conversion of oscillatory *vis viva* into projectile *vis viva*, of linear, spherical and explosive centres of oscillation. One of these relations, which seems specially important and significant in connection with the maintenance of solar energy, is found by dividing the virtual radius of solar torsion ($ar_0 = 688.95r_0$) by Earth's mean radius vector ($\rho_3 = 214.45r_0$). This gives a relative distance of $1 \div 3.2134$, and a relative orbital velocity of $(3.2134 \div 1)^{\frac{1}{2}} = 1.7926 \div 1$ at the chief centre of condensation in the solar system, as compared with the orbital velocity at the extremity of the radius of torsion. This is within about $\frac{5}{12}$ of one per cent. of the ratio (1.8) of the *vis viva* of oscillating particles to the *vis viva* of wave propagation, which was indicated by me in 1872 (*Proc. Am. Phil. Soc.*, xii, 394), and by Maxwell in 1877 (*Phil. Mag.* [5] iii., 453, iv. 209). A portion of this trifling difference may be explained by the gradual diminution of æthereal density upon receding from stellar centres (Note 240).

278. *Collateral Hypotheses.*

In investigating the relations of centripetal and centrifugal action and reaction, it seems desirable to consider and compare the following hypotheses and conclusions :

1. Laplace's estimate that the velocity of transmission, in gravitating acceleration, is at least 100,000,000 times as great as that of light.
2. LeSage's hypothesis that gravitation and luminous undulation represent equal actions and reactions.
3. Faraday's fruitless search for a gravitating constant which would satisfy his interpretation of the doctrine of conservation of energy.
4. Herschel's comparison of the mean *vis viva* of light, with that of sound.
5. Weber's identification of the velocity of light (v_λ), with the "electro-magnetic ratio" (v_ε).
6. Berthelot's "explosive waves," and their action upon sound waves.
7. The inquiries of Siemens into the combined influence of rotation, centrifugal action, gravitating force, and chemical affinity.
8. The various attempts of Thomson, Rankine, Maxwell, Eddy, and others, to escape the apparent consequences of the second law of thermodynamics.

279. *Cosmical and Molecular Constant.*

If there is a natural unit of force we may look for a natural unit of velocity. The hypothesis that all the particles of bodies are in constant motion, involves oscillation of various kinds, orbital, pendulous, or wave. Different transformations of similar oscillations are harmonic. Rotation may be regarded as a pendulous oscillation due to retarded and modified revolution. The resemblance of LeSage's theory to the kinetic theory of gases points to a probability that the natural unit of velocity is oscillatory. This probability is strengthened if we assume the existence of molecular and intermolecular elasticity. In looking to the activities of the principal mass in our system for indications of a natural unit of velocity, we find that gravitating velocities may be represented by gt. In order that gt may be constant, t must vary inversely as g and, therefore, directly as r^2. This variation is found in the rotation of a nebulous sphere, where it holds good for all stages of expansion or contraction which are not affected by external influence. Gravitating acceleration should do its whole work in stellar rotation, as well as in planetary revolution. Particles exposed to solar superficial gravitating acceleration, during a single oscillation of half-rotation, would acquire a constant velocity ($gt = v_\gamma$) which is equivalent to the velocity of light. In other words, the unit of velocity which is indicated by the combined constant of solar gravitation and rotation is the same as is indicated by light and by electricity.

280. *Harmonious Development.*

The velocity of light, like the velocity of sound, represents an elastic atmosphere whose height, if homogeneous, would be twice the virtual fall which would give the velocity in question. The analogy is enhanced by the fact that the hypothetical elasticity of the luminiferous æther is in harmonic accordance with solar rotation and planetary revolution. The identity of velocity

$$v_\lambda = v_\varepsilon = v_\gamma$$

represents three simple relations of light, electricity and gravitation. Interpret this fact as we may, it is strongly suggestive of some interplay between sun and matter outside of it, which has determined solar constants. The invariability of v_γ by nebular contraction indicates the same determination for the present and for all future time. If Sun's mass and magnitude are determined by cosmic causes, so that the solar system can always be considered as undergoing a process of harmonious development, the mass, rotation and other solar elements seem likely to be connected by numerical relations, either with other bodies of the system or with cosmical or physical data.

281. *A Fundamental Time-Integral.*

Thomson and Tait (*Nat. Phil.*, i, § 52) speak of the principle of harmonic motion as one of "immense use, not only in ordinary kinetics, but

in the theories of sound, light, heat, etc." Nearly all the results of my physical investigations go to confirm the truth of this statement. The velocity which is involved in the time-integral of projection against a constant gravitating retardation, is measured by gt. The theory of the ballistic pendulum assumes (*op. cit.*, § 298) "that the ball and pendulum are moving on as one mass *before the pendulum has been sensibly deflected from the vertical.* This is the essential peculiarity of the apparatus. A sufficiently great force might move it far from the vertical in a small fraction of its time of vibration. But in order that the time-integral may have its simplest application to such a case, the direction of the force would have continually to change so as to be always the same as that of the motion of the block."

This is precisely the case in the identity of the foregoing note, according to LeSage's hypothesis. The doctrine of correlation of force leads us to look for the simplest forms of harmonic motion at the centres of stellar systems. The simplest value of t, in a harmony of luminous undulation and stellar rotation, is that of a single oscillation of half-rotation. We have no means of knowing whether the identity, $v\lambda = v_{\Xi} = v\gamma$, holds for any system except our own, but its verification by our sun and the variety of ways in which photodynamic harmonies are deduced are very significant.

282. *A Secondary Time-Integral.*

The harmony between Sun's constrained rotation and luminous undur lation warrants an expectation of subordinate harmonies between sola- and planetary motions. We may naturally look for the simplest relation in some harmonic motions of Sun and Jupiter. Jupiter is at the nebular centre of the system, on a diameter which is bounded by mean loci of Neptune and Uranus, and the velocity which is involved in its time-integral of rotary oscillation, (gt), is nearly, and perhaps exactly, the same as the limit of planetary velocity in a circular orbit ($\sqrt{gr}$ at Sun's surface). Although planetary revolution at Sun's surface is impossible, the influences which tend to produce it are continuous, and any wave motion which may be thus produced is propagated with uniform velocity through the medium in which the waves originate. The uncertainties in regard to the exact values of Earth's semiaxis major and the apparent semidiameter of Sun and Jupiter, introduce a range of uncertainty into the velocity of Jupiter's time integral, which amounts to about five per cent. Values may be taken which are very near the mean values and which make the accordance exact. This accordance may, perhaps, lead to a special extension and modification of George H. Darwin's beautiful investigations.

283. *A Third Time-Integral.*

The centre which seems to hold the third rank in point of cosmical importance, in the solar system, is the centre of the belt of greatest condensation, which is represented by Earth's orbit. The velocity which is in-

volved in its time-integral of rotary oscillation is slightly less than Jupiter's corresponding velocity, being almost, or quite identical with planetary velocity at the mean centre of gravity of Sun and Jupiter. These successive accordances furnish data for a second "photodynamic problem of three bodies," which is, perhaps, even more remarkable than the one given in Note 254. The importance of the combined harmonies may be shown by a simple recapitulation of the several harmonic velocities, viz. : 1. The identity of Note 280 ; 2. The velocities which correspond with the respective time-integrals of rotary oscillation for the chief centre of nucleation (Sun), the centre of nebulosity (Jupiter), and the chief centre of condensation (Earth) ; 3. The limiting velocity of circular orbital revolution in the system ; 4. The velocity of circular orbital revolution at the centre of gravity of Sun and Jupiter.

284. *Instantaneous Action.*

The case of gravitating action and re-action between Moon and Earth (*Thomson and Tait,* § 276), is the one which led Laplace to his highest estimate of the velocity of gravitating transmission and to suppose that the transmission might be absolutely instantaneous. It is also the case which led Adams (*Ib.* § 830) to the discovery of Laplace's error respecting the theoretical invariability of the mean sidereal day and to the subsequent discussions of tidal friction and retardation. That there is such a thing as instantaneous action is so generally believed that it seems desirable that attempts should be made to furnish some physical representation of its possibility and to demonstrate its influence upon adjustments of equilibrium in cosmical actions and reactions. If frictional accelerations in one portion of a rotating globe can be compensated by frictional retardations in another, or if elasticity (Note 217) aids tidal tendency and wave propagation in making the instantaneous changes which are required by tidal action, our tidal theories need careful revision. The *facts* of harmonic relation which are found on all sides, indicate activities which have been at work in all time, and they should not be ignored for any merely theoretical considerations.

285. "*Harmonic Analysis of Tidal Observations.*"

At the last meeting of the British Association, a special grant of £50 was made to Mr. George H. Darwin, for a Harmonic Analysis of Tidal Observations. Mr. Darwin's success in developing Sir William Thomson's views upon cosmical viscosity, and the beauty of many of his results, give assurance of valuable additions to human knowledge from any work that he may undertake. The accuracy of the conclusions which he has already drawn from Delaunay's hypothesis, is unquestionable. My criticisms (Notes 215–225) upon Prof. Ball's use of those conclusions, were based upon the fact that they did not adequately represent all the elements of the questions which were involved, the laws of intermolecular elasticity and harmonic motion having been almost entirely overlooked. In the absence of any positive evidence of tidal retardation, we have no right to

jump at the conclusion that it is established by the second law of thermo-dynamics. The "reproach" which that law involves is increasingly felt by able investigators (Note 278, 8), and even if it should at last be unanimously admitted that the reproach is unavoidable, it is more satisfactory to suppose a continual restoration of energy by divine supervision, than to believe in the spasmodic alternations of rest and activity, which are taught in the Hindoo mythology.

286. *Refraction of Energy.*

The important cosmical time-integrals and the triple identity of fundamental velocities (Notes, 280–3), seem to be indicative of a continual equivalence of centripetal and centrifugal activities, such as LeSage made the basis of his hypothesis; the rotation of stellar centres serving both to maintain the active energies of the universe and to provide cyclical adjustments of equilibrium. The apparent requirements of thermo-dynamics may, perhaps, be partially satisfied by the probability that the æthereal atmosphere of every star has a relatively hot and a relatively cold hemisphere. It seems possible that all radiations, luminous, thermal, electrical, or kinetic in any other form, may be so refracted, in their passage through the various stellar atmospheres, as to be either reflected from star to star, or transiently absorbed by media which can speedily be enabled, by stellar rotation, to give them out again.

287. *Another Phyllotactic Atomic Divisor.*

The di- and tetratomic group of chemical elements can be more nearly represented by the phyllotactic divisor $\frac{2}{3}$ C = 7.9824, than by 8 H (Notes, 271–2).

		T.	O.	T∽O.	(Log. T∽O.)
O	2 D	15.9648	15.9633	.0015	$\bar{3}$.17609
S	4 D	31.9296	31.984	.0544	$\bar{2}$.73560
Se	10 D	79.8240	78,797	1.027	0.01157
Te	16 D	127.7184	127.960	.2416	$\bar{1}$.38310
Mg	3 D	23.9472	23.959	.0118	$\bar{2}$.07188
Ca	5 D	39.9120	39.990	.078	$\bar{2}$.89209
Sr	11 D	87.8064	87.374	.4324	$\bar{1}$.63589
Ba	17 D	135.7008	136.763	1.0622	0.02620
C	2 D	15.9648	11.9736	3.9912	0.60110
Si	4 D	31.9296	28.195	3.7346	0.57224
Ti	6 D	47.8944	49.846	1.9516	0.29039
Zr	11 D	87.8064	89.367	1.5606	0.19329
Sn	15 D	119.7360	117.698	2.038	0.30920
Hg	25 D	199.5600	199.712	.152	$\bar{1}$.18184
Mo	12 D	95.7888	95.527	.2618	$\bar{1}$.41797
W	23 D	183.5952	183.610	.0148	$\bar{2}$.17026
U	30 D	239.4720	238.482	.99	$\bar{1}$.99564
					$\overline{10}$.66435
			17 (log. D = .90213)		15.33621
				17)	$\overline{25}$.32814
Logarithm of mean residual,					$\bar{2}$.54871

288. *Another Basis for Estimates of Probability.*

The substitution of $\frac{2}{3}$ C for $\frac{1}{2}$ O or 8 H, in Notes 271–2, not only introduces another evidence of phyllotactic influence upon atomicity, but it also shows that the organic elements, C, H, O, N, stand in important phyllotactic relations to four fundamental groups of elements. If we omit C from the comparison, the remaining elements of the di- and tetratomic group give $\bar{2}.47682$ for their logarithm of mean residual. The respective residuals themselves are $.03538D$ and $.02998D$. I have already considered various probabilities, which were based on Schuster's estimates, as well as relative probabilities which are independent of any absolute estimates. Another satisfactory basis of comparison may be found in the mean limiting value of the residual, $a \backsim nD = \frac{D}{5.43654} (2\pi D)^{\frac{1}{2D}}$, when the possible residuals are taken in arithmetical progression. If all possible values are thus taken, in other words, if the number of terms is infinite, the second factor becomes unity and the limiting value is $\frac{D}{5.43654} = .18394D$. This is 5.2 times as great as the first of the above mean residuals, or 6.135 times as great as the second.

289. *Resumé of Phyllotactic Atomicity.*

The most satisfactory phyllotactic divisors for the four elementary groups, as indicated by the foregoing notes, are the following: α, for the monatomic group, $H = 1 \doteq \frac{1}{2}.\frac{1}{8}\ O$; β, for the tri- and pentavalent group, $\frac{1}{3}.\frac{1}{3}$ N $= 1.558$; γ, for the di- and tetratomic group, $\frac{2}{3}\ C \doteq \frac{1}{2}$ O $\doteq 8\ H = 7.9824$; δ, for the residuary metallic group, $\frac{1}{2}.\frac{1}{8}\ O \doteq \frac{1}{8}\gamma_{,,} = .998$. The comparative residual percentages, as deduced from Note 272, and from these divisors are given in the table below:

	S_2.	S_1.	H.	Gerber.	Chase.
Monat.	.20117	.20868	.08483	.11623	.08483
3 and 5	.18626	.15532	.09306	.06510	.06499
2 and 4	.24947	.16498	.12405	.04988	.02998
Metal.	.22497	.20023	.14752	.22172	.06882
Periss.	.19432	.18271	.08344	.08955	.07525
Artiad.	.23414	.18584	.13797	.12460	.04965
Mean	.22089	.18483	.12007	.11237	.05654

290. *Comparative Probabilities.*

The following tables give the comparative probabilities for the several divisors: 1st, if the hydrogen unit; 2d, if $.18394D$ is taken as the unit of probability.

	S_2	S_1	H.	Gerber.	Chase.
Monat.	.4217	.4065	1.0000	.7298	1.0000
3 and 5	.4996	.5991	1.0000	1.4295	1.4318
2 and 4	.4972	.7519	1.0000	2.4868	4.1377
Metal.	.6558	.7368	1.0000	.6653	2.1625
Perissad.	.4551	.4840	1.0000	.9876	1.1753
Artiad.	.5893	.7424	1.0000	1.1073	2.7787
Mean	.5436	.6496	1.0000	1.0684	2.1235

Aggregate, S_2 .0000000000000000114; S_1 .00000000000102; H 1; G 69.019; C 853,782,000,000,000,000,000.

Monat.	.9143	.8814	2.1685	1.5825	2.1685
3 and 5	.9876	1.1843	1.9766	2.8255	2.8302
2 and 4	.7373	1.1149	1.4829	3.6876	6.1356
Metal.	.8176	.9187	1.2469	.8296	2.6964
Perissad.	.9466	1.0067	2.0799	2.0541	2.4445
Artiad.	.7856	.9898	1.3332	1.4763	3.7046
Mean.	.8327	.9952	1.5320	1.6369	3.2533

Aggregate, S_2 .00000815; S_1 .735; H 718,725,600,000; G 49,821,227,000,-000; C 614,812,000,000,000,000,000,000,000,000,000.

291. *Another Comparative Basis.*

In the above comparisons it seemed best to exclude the elements that were *exactly* phyllotactic multiples of the assumed divisors (H in the 3d and 5th columns; N and C in the 5th). If those elements were considered as uncertain to the amount of .001 H, the results would be modified, by introducing all the elements, as follows: residuals; H, monatomic, .05859, perissad, .07144, mean, .11154; Chase, monatomic, .05859, tri- and pentatomic, .03891, di- and tetratomic, .03538, perissad, .04916, artiad, .05293, mean, .05168. The comparative mean probabilities would be as follows: S_2 .5050, S_1 .6035, H 1.0000, Gerber .9926, Chase 2.1581. The mean probability of the hydrogen unit, as deduced from the mean accidental residual, would be, 1.9286; of the phyllotactic divisors, 3.5589. That the test of the mean accidental residual is sufficiently severe is evident from the probabilities which it indicates for the surd divisors, S_1 and S_2.

292. *Objection Answered.*

The uncertainity, even of Clarke's recomputation of atomic weights, has been urged as an objection to the acceptance of any apparent probabilities which may be inferred from their examination. If our conclusions were absolute, the objection would be valid, and it must be admitted that even the comparative probabilities will doubtless be greatly modified by the more accurate determination of doubtful atomicities. The modifications, however, would be quite as likely to increase the evidences of phyllotactic influence as to diminish them, if there were no reason to look for such influence. The *a priori* grounds for expecting proof of harmonic action in some shape or

other (Note 281), together with the various physical tendencies to division in extreme and mean ratio (Notes 135, etc.), which make phyllotaxy a simple form of harmony, seem likely to turn the scale largely on the side of its present leaning, so as to make the fact of atomic phyllotaxy more and more evident with each successive increase of precision in atomic measurements. While the mean probability of the hydrogen unit, under the most favorable aspect, is 1.93 times as great as that of any divisor taken at random, the mean probability of the phyllotactic divisors, under the least favorable aspect, is 1.845 times as great as that of hydrogen. If my divisors, like Gerber's, had been purely empirical, there would have been more reason to think that they might lose credit with increased precision of determination, but even then it would be strange if so large a relative advantage were entirely overcome. The successive discoveries that Gerber's divisors are approximately phyllotactic, that their significance is increased by making them exactly phyllotactic, and that the most satisfactory divisors which have yet been found stand in simple phyllotactic relations to the four fundamental organic elements, furnish no ground for expecting any future reversal or weakening of the harmonic indications which I have already set forth.

293. *Photodynamic Precession.*

To the many harmonic evidences of photodynamic action and reaction between the chief centres of nucleation and of condensation, Sun and Earth, may be added one which serves to illustrate and extend the principles that are involved in my first "photodynamic problem of three bodies" (Note 254). If we suppose the photodynamic rotating influence on the æthereal sphere, at the equatorial locus of Sun's modulus of light ($474028r_0$; Note 263), to be such as would give planetary velocity at the same locus, the time of rotation would be $(474028 \div 214.73)^{\frac{3}{2}} = 103721$ years. If nebular condensation were to begin at that locus and proceed until the primitive velocity of the locus would tend, through viscosity, to become parabolic, the nucleal radius would be reduced to one-half and the time of rotation to one-fourth of the primitive values. The period, or "great year," which is thus indicated (25930.25 years), is virtually identical with a complete revolution of the equinoxes, which Herschel estimates at 25,868 years; Stockwell at 25,694.8 $\pm$ 281.2 years;* Newcomb and Holden "about 25,800 years." This accordance furnishes another reason for believing, with Laplace, in the stability of the physical universe, rather than in the ultimate stagnation which seems to be indicated by the questionable second law of thermodynamics.

294. *Harmonic Rotation of Earth and Moon.*

The improbability of Delaunay's hypothesis is further increased by harmonies of rotation which involve the conjoint action of Sun, Earth and Moon.

* The differences from the mean value being due to secular inequalities.

By taking the rotating locus of the linear centre of oscillation, for Laplace's terrestrial limit, l, we find that the velocity of rotation at $\frac{1}{3}\,l$ is virtually identical with Moon's mean velocity of revolution. Let $l = nr_3$; then $t_a = 2\pi\sqrt{\frac{n^3 r_3}{g_3}} = 86164.1$ seconds; $n = 6.60704$; $\frac{1}{3}\,l = 2.20235 r_3$; velocity of rotation at $\frac{1}{3}\,l = 4.40469\ \pi r_3$ per sidereal day, or $4.41675\ \pi r_3$ per mean solar day. If this is Moon's mean orbital velocity, the circumference of her orbit is $(27.321661 \times 4.41675 = 120.673)\ \pi r_3$. Moon's orbital eccentricity being .0549081, her orbit is $.999246 \times 2\pi a$ and $a = 60.382\ r_3$. Proctor's estimate is $60.263\ r_3$; Littrow's, $60.278\ r_3$; Newcomb's $60.639\ r_3$. See, also, Note 296.

295. *Spectrum of Comet Wells.*

Huggins (*Nature*, June 22, 1882, p. 179) gives a band spectrum, with measured wave-lengths for the brightest portions. Its harmonies are shown in the following comparisons:

	Huggins.	Divisors.	Harmonic.
α	4769	1	4769
β	4634	$1 + a$	4634.2
γ	4507	$1 + 2a$	4507.2
δ	4412	$1 + 2b$	4412.1
ε	4253	$1 + 3b$	4252.9

$$\beta - \gamma : \gamma - \varepsilon :: 1 : 2$$
$$\gamma - \varepsilon : \beta - \varepsilon :: 2 : 3$$

In other words, γ is the centre of linear oscillation between ε and β.

Other phyllotactic approximations are indicated by the proportions:

$$\delta - \varepsilon : \gamma - \varepsilon :: 5 : 8 \text{ nearly.}$$
$$\gamma - \varepsilon : \alpha - \varepsilon :: 1 : 2 \quad \text{“}$$

These several relations show a primitive phyllotactic tendency, which is controlled and modified by γ and the harmonic divisors. The following values would *exactly* satisfy all the phyllotactic harmonies: 4760.71, 4633.86, 4507, 4411.86, 4253.29.

296. *Harmonic Nebular Time-Integrals.*

The second "photodynamic problem of three bodies," which is specially implied in my three primitive time integrals (Notes 281–3), may be associated with the first through a harmonic relation which involves Moon's orbital time (t_β), Earth's rotation (t_α), Earth's superficial gravitating acceleration (g_3), and Sun's gravitating acceleration at the perihelion centre of gravity of Sun and Jupiter (g_0). The relation is expressed by the proportion.

$$t_\beta : t_\alpha :: g_0 : g_3$$

The resulting equation, $g_3 t_\beta = g_0 t_\alpha$, indicates two important harmonic time-integrals, which seem much more likely to be permanent, than to be

disturbed and even overthrown by tidal friction and retardation. Since g is taken at the present locus of Jupiter's orbital projection, it seems possible that the lunar disturbance, which Delaunay referred to tidal friction, may have a secular period, which represents some function of Jupiter's secular variations of eccentricity. If we take Leverrier's estimate of Jupiter's present eccentricity, .0482388, and Stockwell's estimate of its secular variation, .0608274, Sun's superficial gravitating acceleration is $1.027\ g_0 = 1.027 \times 27.321661 = 28.059$. This gives $\rho_3 = 92,409,000$ miles, if we take the oscillatory estimate of Solar mass, and the British Nautical Almanac estimate of Sun's apparent semi-diameter ($m_0 = 331,776\ m_3$; $\rho_3 = 214.45 \rho_0$). Compare Note 256, *e*.

297. *Two Tidal Questions.*

No physical question can be regarded as satisfactorily settled, until all the known facts which are likely to have any bearing on its solution have been duly considered. Provisional hypotheses may be very properly adopted as occasional and temporary expedients, in order to fix new points of departure, and facilitate the progress of investigation, but even they are defective whenever they are obviously limited and partial. The cosmical importance of harmonic motion, which Laplace demonstrated in his discussions of Jupiter's satellite system, as well as the further evidences of its general physical importance which have been brought forward by Lagrange, Fourier and Thomson, cannot be wisely set aside, even in a provisional hypothesis, through any dogmatic assertion of a thermodynamic requirement, which, if it is not compensated in some way, may possibly lengthen the terrestrial day by a minute interval, which has been variously estimated, from $\frac{1}{80000}$ to $\frac{11}{50}$ of a second in a year. Even if the requirement was universally admitted, the relations of photodynamic precession (Note 293), indicate a possible harmonic acceleration which is manifoldly greater than this problematical retardation. Before making any admission which would call for a careful study of this possible acceleration, two questions should be satisfactorily answered: 1. Are the tidal tendencies instantaneously adjusted? 2. Are the local tidal frictions limited to mere terrestrial action, so that the conversion of motion into heat, at one point, is compensated by a conversion of heat into motion at another?

298. *Explosive Waves.*

Berthelot's discovery has already been suggested (Note 278, 6) as one of the important topics for consideration in the study of æthereal correlations. The velocity, $\sqrt{gh}$, which is indicated by the explosive energy of H_2O (Note 16), is $(32.088 \times 68878.2 \times 1389.6 \div 9)^{\frac{1}{2}} = 18473$ ft. $= 3.49865$ miles per second. This velocity is sufficient, under the normal atmospheric pressure at Earth's surface, to produce æthereal waves which are manifested by light, heat and chemical combination. We may accordingly look for like phenomena whenever "subsiding" particles penetrate the

nebulous region of the zodiacal light with a corresponding *vis viva*. Subsidence from Laplace's solar limit (Notes 268, 274), would give a *vis viva* which is more than 10000 times as great, in their passage through the solar atmosphere. These facts should be carefully considered in any investigations which are suggested by the hypothesis of Dr. Siemens. The explosive velocity being acquired long before the subsiding matter reaches Sun's surface, the compounded and condensed particles continue sunward into the region of dissociation and centrifugal projection. No sufficient reason has yet been given, for doubting the adequacy of the fundamental time-integral (Notes 280–1) to keep up this circulation indefinitely. Important harmonic analogies are suggested by Neptune's projectile orbital velocity at secular perihelion, and by Jupiter's mean locus of subsidence. According to Stockwell's estimates of the planetary elements, Neptune's secular perihelion velocity is 3.42 miles per second and Jupiter's mean aphelion is 5.4274 ρ_3; the mean proportional between Earth's semi-axis major and Neptune's secular perihelion being 5.4404 ρ_3.

299. *Alternations of Energy.*

All the ordinary assumptions of dissipation of energy take it for granted that the universal æther is able to absorb heat indefinitely, without imparting it again to more condensed matter. If this were the case, why should not the heat be absorbed in its passage from star to star? Judging from atmospheric analogies, we may infer the existence of æthereal convection currents and a greater manifestation of heat with increasing density. If æthereal density varies with pressure, as I have supposed in Notes 35, 236–240, etc., the kinetic theory of gases would imply a constant mean molecular velocity. The tangential character of luminous undulations implies a polarity which would tend to the formation of æthereal spheroids about stellar centres, and if those centres have an orbital motion which is combined with an axial rotation of their respective orbs, the continual changes of relative position would favor a transfer of energy from star to star which, with reflection and refraction (Note 286), might maintain perpetual tendencies to an equilibrium which would never be reached. It seems not unlikely that the thermal relations of every star to its æthereal spheroid may be so adjusted that there is a transfer of heat from the æther to the nucleus during one-half of each rotation, and from the nucleus to the æther during the other half. Such a hypothesis lends a meaning to the fundamental kinetic identity (Note 280), which is in thorough accordance with Laplace's belief in the stability of the solar system.

300. *Actions and Reactions in Moving Radiations.*

Prof. H. T. Eddy (*Sci. Proc.* of the *Ohio Mech. Inst.*, July, *1882*) describes a method for the distribution of heat in a way which conflicts with the second law of thermodynamics. He objects to the so-called axioms of Clausius and Thomson, on the ground of their implicit assumption

that heat is radiated with infinite velocity, inasmuch as they take no account of the states of relative rest or motion of the bodies between which the heat passes. He cites the statement of Kirchhoff, "that the second law cannot be (at present) proved ; but it, so far, has never been found in disagreement with experience;" the view of Maxwell and Boltzmann (*Wien Sitzb.*, Bände, lxxvi, lxxviii), that it should be regarded "as merely the mean result flowing from the laws of probability ;" Rankine's paper (*Phil., Mag*, [4] iv, 358), in which "he has supposed it possible to reflect radiations in such a way as to give the universe such differences of temperature as to insure it a new lease of life ;" and the paper of Clausius (*Mech. Theory of Heat*, chap. xii), showing the general impossibility of such a reconcentration as Rankine supposed, when the radiating bodies are at rest ; nevertheless, no such impossibility may finally appear in case of the actual universe which is a system of moving bodies." He closes his discussion with the following sentences : "The point to which I would emphatically direct attention is, that since radiations are known to be moving in space, apart from ponderable bodies, and subject to reflections, it is possible so to deal with them as to completely alter their destination, and successfully interfere with all results flowing from Prevost's law of exchanges. It also seems to me that the exactness of the second law of thermodynamics depends, as far as radiations are concerned, upon that of this law of exchanges." In addition to the reflections to which moving radiations are subject, I have also called attention to their refraction (Note 286), and I have endeavored to co-ordinate all my discussions, through the fundamental identity (Note 280), which implies an equivalent motion of reaction for every radiant action. Moreover, the moving particles in each radiant undulation are all subject to cosmical attractions and perturbations, which have not yet been considered in investigations of the seeming dissipation of energy.

301. *Thrust of Polar Ice-Caps.*

Geologists who believe that the northern hemisphere was once largely covered with ice, have usually attributed the thrust to the simple gravitating pressure of the accumulation at the pole. The position of many of the boulders, and of the supposed terminal moraines, seems to indicate a greater propelling force than many investigators are willing to attribute to the combined action of polar centripetal and equatorial centrifugal energy. Perhaps the unwillingness may be removed by making proper allowance for "the flow of solids," an element of the problem which does not seem to have received any consideration beyond the simple plasticity and regelation which have been studied in connection with the movements of ordinary glaciers. The photodynamic hypothesis of an all-pervading and universally active æther involves the requirement of perpetual tendencies toward equilibrium, and the evidence of such tendencies which is given by Earth's oblateness (Notes 246, 249) furnishes an adequate explanation for many of the glacial phenomena which have hitherto seemed paradoxical. Bessel's estimate of the oblateness is slightly less than would result

from Tresca's "flow;" Clarke's two estimates accord more nearly with the theoretical value; while Listing's, which is the latest of all, gives an agreement which is virtually exact. If we start from his estimate (1 : 288.4), we get $g = \frac{4\pi^2 \times 288.4}{(86164.1)^2} r = 32.086$ ft. Ganot's value is 32.088 ft. It can hardly be believed that such a coincidence is merely accidental. If it is indicative, as I have supposed, of inter-molecular æthereal action, it has an important bearing on tidal equilibrium, and it shows that Earth's shape and rigidity were not fixed in any past age, but are at all times adjusted to the requirements of internal elasticity and external attractions. Any arguments which may be adduced in favor of such an adjustment may be urged, *a fortiori*, in support of the flow and thrust of a plastic material like ice. The velocity of terrestrial rotation, in the mean latitude which Prof. H. C. Lewis has indicated for the terminal moraine in Pennsylvania, is more than 1000 feet per second. The centrifugal force consequent upon such a velocity, together with the thrust of an ice-cap which extended to the pole, must greatly facilitate glacial flow. The equilibrating forces would work upon local glaciers, in the same way as upon a general ice-cap.

The Classification of the Ungulate Mammalia. By E. D. Cope.

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In the present essay the osseous system is chiefly considered, and of this, the structure of the feet more than of any other part of the skeleton. The ungulata are here understood to be the hoofed placental Mammalia with enamel covered teeth, as distinguished from the unguiculate or clawed, and the mutilate or flipper limbed, and the edentate or enamelless, groups. The exact circumscription and definition is not here attempted, though probably the brain furnishes an additional basis of it in the absence of the crucial, parietoöccipital, calcarine fissures, etc. Suffice it to say that it is on the whole a rather homogeneous body of mammalia, especially distinguished as to its economy by the absence of forms accustomed to an insectivorous and carnivorous diet, and embracing the great majority of the herbivorous types of the world.

The internal relations of this vast division are readily determined by reference to the characters of the teeth and feet, as well as other less important points. I have always insisted that the place of first importance should be given to the feet, and the discovery of various extinct types has justified this view. The predominant significance of this part of the skeleton was first appreciated by Owen, who defined the orders *Perisso-*